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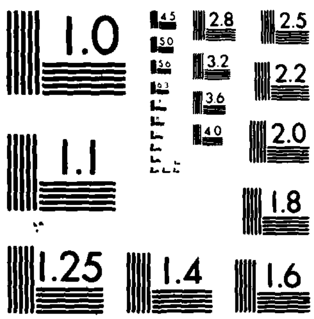

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VIBRATION OF CYLINDRICAL SHELLS OF BIMODULUS COMPOSITE MATERIALS

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Abstract

A theory is formulated for the small-amplitude free vibration of thick, circular cylindrical shells laminated of bimodulus composite materials, which have different elastic properties depending upon whether the fiber-direction strain is tensile or compressive. The theory used is the dynamic, shear deformable (moderately thick shell) analog of the Sanders best first-approximation thin-shell theory. By means of tracers, the analysis can be reduced to various simpler shell theories, namely Love's first approximation, and Donnell's shallow-shell theory. As an example of the application of the theory, a closed-form solution is presented for a freely supported panel or complete shell. To validate the analysis, numerical results are compared with existing results for various special cases. Also, the effects of the various shell theories, thickness shear flexibility, and bimodulus action are investigated.

Index categories: Structural Dynamics; Structural Composite Materials.

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Nomenclature

A_{ij} = stretching stiffnesses

B_{ij} = bending-stretching coupling stiffnesses

b = circumferential arc width of panel

$[C]$ = matrix of coefficients

C_ℓ, C'_ℓ = shell-theory tracers

$\bar{C}_\ell$ = C_ℓ/R

D_{ij} = bending stiffnesses

d_{xy} = $\partial^2(\quad)/\partial x \partial y$

h = total shell thickness

I = rotatory inertia coefficient (per unit midsurface area)

K_4, K_5 = thickness-shear correction factors

$[L]$ = matrix of linear differential operators

M_i, N_i = stress couples and in-surface stress resultants

m, n = axial and circumferential mode numbers

P = normal inertia coefficient (per unit midsurface area)

Q_i = thickness-shear stress resultants

$Q_{ijk\ell}$ = plane-stress reduced stiffness coefficients

R = radius of shell midsurface

S_{ij} = thickness-shear stiffnesses

t = time

u, v, w = axial, circumferential, and radial midsurface displacements

U, V, W = amplitudes of u, v, w

x, y, z = position coordinates in axial, circumferential, and outward normal directions

X, Y = amplitudes of ψ_1 and ψ_2

z_{nx}, z_{ny} = neutral-surface positions for ε_x=0 and ε_y=0, respectively

Z_x, Z_y = z_{nx}/h, z_{ny}/h

α, β = $m\pi/L$, $n\pi/b$ (for panel) or n/R (for complete cylinder)

δ, Δ = generalized displacement vectors defined in Eqs. (8) and (15)

ψ_1, ψ_2 = bending slopes in xz and yz planes

ε_j = strain components at arbitrary location (x,y,z)

ε_j^0 = midsurface strain components

κ_j = curvature-change components

ρ = material density

σ_i = stress components

ω = natural frequency

$(\ ),_{xy}$ = $\partial^2(\ )/\partial x \partial y$

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Introduction

Analysis of composite-material shell-type structures has been of considerable research interest in the past two decades. Earlier work has been summarized in a monograph by Leissa[1] and in two more recent survey papers[2,3].

Due to its importance, the finite-length circular cylindrical shell configuration has received the most attention. Thin-shell analyses of composite-material shells of this configuration have been performed by Dong[4], Abhat and Wilcox[5] and Fortier and Rossettos[6] using Donnell's shallow-shell theory[7], by Bert, Baker, and Egle[8], Stavsky and Loewy[9], Shivakumar and Krishna Murty[10], and Greenberg and Stavsky[11] using Love's first approximation theory[12], and by Padovan[13] using Novozhilov's higher-order theory[14]. In the present work, tracer coefficients are introduced to consider Sanders' best first-approximation theory[15], Love's first approximation[12], Loo's[16], Morley's[17], and Donnell's[7] theories.

The thickness shear moduli of composite materials are typically an order of magnitude lower than their in-plane elastic moduli. This is in contrast to isotropic materials, which have shear to elastic moduli ratios of approximately 40%. Thus, composite-material shells would be expected to be affected by thickness-shear flexibility to a much greater extent than isotropic shells having the same geometric parameters. Previous research on vibration of laminated, shear deformable shells includes that of Refs. 18 and 19, using the shear deformable analog of Donnell's shallow shell theory, and Refs. 20 and 21, using the shear deformable version of Love's first approximation. In some recent work[22], the shear deformable analog of Sanders' theory was developed and it was shown, using tracers, that this theory can be reduced to two simpler shear deformable theories as special cases: (1) Love's first approximation and Loo's, and (2) Morley's and Donnell's. This approach is extended to shell vibration in the present paper.

Certain fiber-reinforced materials, especially those with very soft matrices (e.g., cord-rubber), exhibit quite different elastic behavior depending upon whether the fiber-direction strain is tensile or compressive[23-25]. As a first approximation, the stress-strain behavior of such materials is usually modeled as being bilinear, with different elastic coefficients depending upon the sign of the fiber-direction strain. Thus, these materials are called <u>bimodulus</u> composite materials. The fiber-governed, symmetric-compliance material model proposed in Ref. 26 has been shown to agree well with experimental data obtained under in-plane loading of several materials with drastically different properties in tension and compression. To the best of the present investigators' knowledge, all of the previous investigations of laminated bimodulus shells, as reviewed in Refs. 22 and 27, have involved

the static behavior of thin cylindrical shells under mechanical or thermal loading. However, free vibration of cross-ply laminated plates was considered in Ref. 28.

Formulation

Laminate Action

To explain the effect of bimodulus action on the shell behavior, we discuss first a flat plate. If a single-layer plate or a laminate symmetrically laminated about its midplane are made of ordinary (not bimodulus) materials, there is no coupling between bending and stretching. However, a laminate which is not symmetrically laminated does exhibit bending-stretching coupling.

Now consider a flat plate consisting of either a single layer of bimodulus material or symmetrically laminated of bimodulus materials. When the plate is subjected to bending, the different elastic moduli in tension and compression cause a shift in the neutral surface away from the geometric midplane toward the tension side of the bend if the tensile properties exceed the compressive ones. Consequently, this plate displays bending-stretching coupling, analogous to a plate unsymmetrically laminated of ordinary material as described in the preceeding paragraph. Therefore, in a laminated bimodulus-material shell, bending-stretching coupling can be induced by the combination of bimodulus-material action, laminate configuration, and shell geometry (curvature).

One can write the laminate constitutive relations as

$$\begin{aligned} \left\{ \begin{array}{c} N_i \\ \hline M_i \end{array} \right\} &= \left[\begin{array}{c|c} A_{ij} & B_{ij} \\ \hline B_{ij} & D_{ij} \end{array} \right] \left\{ \begin{array}{c} \epsilon_j^0 \\ \hline \kappa_j \end{array} \right\} \qquad (i,j=1,2,6) \\ \{Q_i\} &= [S_{ij}]\,\{\epsilon_j\} \qquad (i,j=4,5) \end{aligned} \tag{1}$$

Here, the respective in-plane, bending-stretching, bending or twisting, and thickness-shear stiffnesses are defined as

$$(A_{ij},B_{ij},D_{ij}) = \int_{-h/2}^{h/2} (1,z,z^2)Q_{ijk\ell}\,dz \qquad (i,j=1,2,6)$$
$$S_{ij} = \int_{-h/2}^{h/2} K_i K_j\, Q_{ijk\ell}\,dz \qquad (i.j=4,5) \tag{2}$$

Here, $Q_{ijk\ell}$ denotes the plane-stress reduced stiffness defined by

$$\{\sigma_i\} = [Q_{ijk\ell}]\{\epsilon_j\} \tag{3}$$

and subscripts ij refer to the position in the stress-strain-relation array, k refers to the sign of the fiber-direction strain ($1 \sim +$, $2 \sim -$), and ℓ is the layer number.

Shear Deformable Shell Theory

The coordinate system used here is shown in Fig. 1. Then the equations of motion in the absence of body forces and body moments can be written as

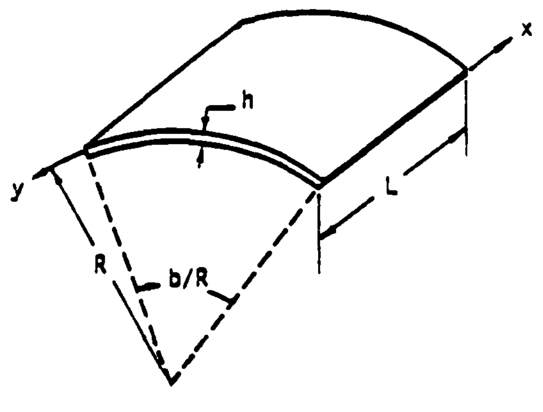

Figure 1. Shell geometry

$$
\begin{aligned}
&N_{1,x} + N_{6,y} - (C_2/2R)M_{6,y} = Pu,_{tt} + (I/R)\psi_1,_{tt} \\
&N_{6,x} + N_{2,y} + (C_1/R)Q_4 + (C_2/2R)M_{6,x} = Pv,_{tt} + (I/R)\psi_{2,tt} \\
&Q_{5,x} + Q_{4,y} - (N_2/R) = Pw,_{tt} \qquad (4) \\
&M_{1,x} + M_{6,y} - Q_5 = I\psi_{1,tt} + (I/R)u,_{tt} \\
&M_{6,x} + M_{2,y} - Q_4 = I\psi_{2,tt} + (I/R)v,_{tt}
\end{aligned}
$$

Here, it has been assumed that the mass distribution is symmetric about the middle surface. The normal and rotatory inertias are

$$(P,I) = \int_{-h/2}^{h/2} (1,z^2)\rho\, dz \qquad (5)$$

The first terms on the right side of each of Eqs. (4) are the same as for plates in Mindlin's shear deformable theory[29]. The second terms on the right side of all except the third equation of set (4) are the inertia-coupling terms due to shell curvature, first introduced by Mirsky and Herrmann[30].

The kinematics of deformation can be expressed as

$$
\begin{aligned}
&\varepsilon_1^0 = u,_x \quad ; \quad \varepsilon_2^0 = v,_y + (w/R) \quad ; \quad \varepsilon_6^0 = u,_y + v,_x \\
&\varepsilon_4 = \psi_2 + w,_y - (C_1/R)v \quad ; \quad \varepsilon_5 = \psi_1 + w,_x \qquad (6) \\
&\kappa_1 = \psi_{1,x} \quad ; \quad \kappa_2 = \psi_{2,y} \quad ; \quad \kappa_6 = \psi_{1,y} + \psi_{2,x} + (C_2/2R)(v,_x - u,_y)
\end{aligned}
$$

The coefficients C_ℓ appearing in Eqs. (4) and (6) are shell-theory tracers which take on values listed in Table 1. Note that in contrast to the case of thin-shell theory, Loo's approximation offers no simplification over Love's first-approximation theory and Donnell's theory coincides with Morley's.

Table 1. Shell-theory tracers and their values

Theory (thin-shell theory generalized to shear deformable theory)	C_1	C_2
Sanders'[15]	1	1
Love's first approximation[12] and Loo's[16]	1	0
Morley's[17] and Donnell's[7]	0	0

Substituting Eqs. (1) and (6) into Eqs. (4), one obtains the following operator equation

$$[L]\{\delta\} = 0 \tag{7}$$

where

$$\{\delta\} = \{u, v, w, \psi_1, \psi_2\}^T \tag{8}$$

and [L] is a symmetric matrix of linear differential operators listed in Appendix A.

Criteria for Homogeneity Along the Middle Surface

In deriving Eq. (7), we tacitly assumed that the laminate stiffnesses ($A_{ij}, B_{ij}, D_{ij}, S_{ij}$) are all independent of coordinates (x,y) on the middle surface. However, in view of the bimodulus nature of the materials comprising the laminate, these stiffnesses depend upon the fiber-direction neutral-surface positions associated with the respective layers (i.e., z_{nx} for a single layer with axially oriented fibers, and z_{nx} and z_{ny} for a cross-ply laminate).

Thus, for layers having the fibers oriented axially, the associated fiber-direction neutral-surface position is determined by

$$\varepsilon_1 = \varepsilon_1^0 + z_{nx}\kappa_1 = 0$$

or

$$z_{nx} = -\varepsilon_1^0/\kappa_1 = -u_{,x}/\psi_{1,x} = \text{constant} \tag{9}$$

Similarly, for layers having the fibers oriented circumferentially

$$\varepsilon_2 = \varepsilon_2^0 + z_{ny}\kappa_2 = 0$$

or

$$z_{ny} = -\varepsilon_2^0/\kappa_2 = -(v_{,y} + R^{-1}w)/\psi_{2,y} = \text{constant} \tag{10}$$

Closed-Form Solution

We seek a solution which satisfies the governing operator Eq. (7), the homogeneity relations (9) and (10), and appropriate boundary conditions.

A closed-form solution is presented for a shell that is freely supported (simply supported without in-surface restraint; SS3 in Hoff's notation[31]) along its curved edges:

$$\begin{aligned}
&N_1(0,y,t) = N_1(L,y,t) = M_1(0,y,t) = M_1(L,y,t) = 0 \\
&w(0,y,t) = w(L,y,t) = v(0,y,t) = v(L,y,t) = 0 \\
&\psi_2(0,y,t) = \psi_2(L,y,t) = 0
\end{aligned} \tag{11}$$

The shell may be either circumferentially complete (closed), which requires that the circumferential mode shapes be periodic in position (i.e., trigonometric functions), or a cylindrically curved panel (open shell) freely supported along the generators, which requires that the following additional boundary conditions be satisfied:

$$\begin{aligned}
&N_2(x,0,t) = N_2(x,b,t) = M_2(x,0,t) = M_2(x,b,t) = 0 \\
&w(x,0,t) = w(x,b,t) = u(x,0,t) = u(x,b,t) = 0 \\
&\psi_1(x,0,t) = \psi_1(x,b,t) = 0
\end{aligned} \tag{12}$$

Under these conditions, the solution to Eq. (7) is of the form

$$
\begin{aligned}
u &= U \cos \alpha x \sin \beta y \cos \omega t \\
v &= V \sin \alpha x \cos \beta y \cos \omega t \\
w &= W \sin \alpha x \sin \beta y \cos \omega t \\
\psi_1 &= X \cos \alpha x \sin \beta y \cos \omega t \\
\psi_2 &= Y \sin \alpha x \cos \beta y \cos \omega t
\end{aligned}
\tag{13}
$$

where $\alpha = m\pi/L$ and $\beta = n\pi/b$ for a panel and n/R for a complete cylinder.

Substitution of Eqs. (13) into Eq. (7) leads to the following homogeneous algebraic system

$$[C]\{\Delta\} = \{0\} \tag{14}$$

where [C] is a matrix with elements depending upon A_{ij}, B_{ij}, D_{ij}, S_{ij}, P, I, α, and β, and

$$\{\Delta\} = \{U,V,W,X,Y\}^T \tag{15}$$

Numerical Results

Before proceeding to the calculations for shells laminated of bimodulus materials, we present some results for both circumferentially complete cylinders and cylindrically curved panels laminated of ordinary composite materials.

The first example is for a thin-walled complete cylinder cross-plied of boron-epoxy. The geometric parameters and material properties used are listed in Table 2. The agreement between the natural frequencies for numerous modes computed by use of the thin-shell theory of Appendix B (specialized to Love's first approximation) and the Love's first-approximation analysis of Ref. 8 is excellent, as shown by comparison of columns 3 and 4 in Table 3. The inclusion of thickness shear deformation sometimes

decreases the natural frequencies, but in other instances increases them, as can be seen in column 5 of Table 3. For this thin shell, the effect of rotatory inertia is almost negligible (column 6).

The second example is for a cylindrically curved panel with thickness shear flexibility included, using numerical input data from Refs. 18 and 19 as listed in Table 4. In order to make a fair comparison with the results of Refs. 18 and 19, it is necessary to use the shear deformable version of Donnell's theory and to neglect rotatory inertia and the tangential inertias ($Pu,_{tt}$ and $Pv,_{tt}$) as suggested by Vlasov[32]. Dimensionless natural frequencies for various modes, with and without bending-stretching coupling (B_{ij}) suppressed, are listed in Tables 5 and 6. The lack of good agreement in Table 5 is difficult to explain, especially in view of the good agreement in Table 6.

In studying the above results, one can discern these general trends:

1. For a complete shell at a fixed axial wave number, the natural frequencies first decrease then increase with increasing circumferential wave number (n). As was first explained by Arnold and Warburton[33], this is due to the decrease in membrane strain energy and the increase in bending strain energy as n is increased.

2. The inclusion of thickness shear deformation and rotatory inertia does not always decrease the frequency. A similar observation may be seen in the recent work of Greenberg and Stavsky[34].

3. Bending-stretching coupling ($B_{ij} \neq 0$) always lowers the natural frequency compared to the value for the uncoupled case ($B_{ij} = 0$).

4. If the frequencies are normalized with the panel width (b), as in Table 6, increasing the aspect ratio (L/b) lowers the frequency values, just as it does in the case of a flat panel (plate).

To illustrate the effects of bimodulus composite-material action, aramid-rubber properties, determined primarily from the experiments of Patel et al.[24] by use of the material model of Ref. 26, are used. Some details of the estimation of certain properties not directly measured in Ref. 24 are given in Ref. 28. The complete set of properties used is listed in Table 7. This material has the most drastic difference between tensile and compressive properties known to the present investigators; thus, it is definitely a worst case.

Although the present problem is a linear one in each portion of a cycle of vibration, it actually is different in the two portions of a cycle. This phenomenon was explained for the vibration of single-layer and two-layer, cross-ply plates in Ref. 28. Here, for completeness, we discuss the cross-ply case: side cross-sectional views are shown in Fig. 2. The inner layer is oriented circumferentially (y direction) and the outer layer is oriented axially (x direction). During the first portion of a cycle, consider the shell to be deflected as shown in Fig. 2(a). Then the neutral surface for ε_x falls outside of the interface within the axially oriented layer, while the neutral surface for ε_y lies inside the interface, i.e., within the circumferential layer. In the latter portion of the vibration cycle, Fig. 2(b), the ε_x neutral surface falls outside the axial layer, and the ε_y neutral surface lies outside of the circumferential layer. Therefore, in the latter portion of the cycle, compressive properties are used for the entire axial layer and tensile properties for the entire circumferential layer. From the foregoing considerations for a two-layer, cross-ply shell, it is clear that the stiffnesses acting in the two portions of a cycle, (a) and (b) in Fig. 2, are different and thus the associated frequencies

(denoted by ω_1 and ω_2, respectively) are also different. The corresponding time intervals over which the two portions take place are π/ω_1 and π/ω_2. Thus, the total period for a complete cycle is $(\pi/\omega_1) + (\pi/\omega_2)$. The average frequency (ω) over the entire cycle is 2π divided by the total period; then

$$\omega^{-1} = (1/2)(\omega_1^{-1} + \omega_2^{-1}) \tag{16}$$

As discussed above, the neutral-surface positions are different during the two portions of a cycle; numerical results are listed in Table 8. The average frequency, as calculated from Eq. (16), is plotted in Fig. 3 versus aspect ratio for various values of R/h. The dashed line shows the analogous results from Ref. 28. for flat plates.

As was mentioned in Ref. 28, it can be shown that in the bilinear analysis presented, energy is conserved at the junction between the two portions of the vibrational cycle.

Concluding Remarks

Two analyses have been presented for prediction of the free-vibrational behavior of freely supported, cylindrically curved panels or complete cylindrical shells cross-ply laminated of bimodulus composite materials. One analysis is for thin shells and the other is for moderately thick shells with thickness shear deformation and rotatory inertia included. When reduced to the special case of ordinary material, the solutions yielded results which agreed well with various existing numerical results. The peculiarities of bimodulus shells have been discussed and the effects of a limited number of parameters investigated. These results may be used to validate numerical analyses, such as those based on the finite-element method.

Acknowledgments

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Appendix A: Linear Differential Operators Appearing in Eq. (7)

$$L_{11} = A_{11}d_x^2 + (A_{66} - \bar{C}_2B_{66} + \frac{1}{4}\bar{C}_2^2D_{66})d_y^2 - Pd_t^2$$

$$L_{12} = (A_{12} + A_{66} - \frac{1}{4}\bar{C}_2^2D_{66})d_xd_y$$

$$L_{13} = (A_{12}/R)d_x \quad ; \quad L_{14} = B_{11}d_x^2 + (B_{66} - \frac{1}{2}\bar{C}_2D_{66})d_y^2 - (I/R)d_t^2$$

$$L_{15} = (B_{12} + B_{66} - \frac{1}{2}\bar{C}_2D_{66})d_xd_y$$

$$L_{22} = (A_{66} + \bar{C}_2B_{66} + \frac{1}{4}\bar{C}_2^2D_{66})d_x^2 + A_{22}d_y^2 - \bar{C}_1^2S_{44} - Pd_t^2$$

$$L_{23} = (R^{-1}A_{22} + \bar{C}_1S_{44})d_y \quad ; \quad L_{24} = (B_{12} + B_{66} + \frac{1}{2}\bar{C}_2D_{66})d_xd_y$$

$$L_{25} = (B_{66} + \frac{1}{2}\bar{C}_2D_{66})d_x^2 + B_{22}d_y^2 + \bar{C}_1S_{44} - (I/R)d_t^2$$

$$L_{33} = -S_{55}d_x^2 - S_{44}d_y^2 + (A_{22}/R^2) + Pd_t^2$$

$$L_{34} = (R^{-1}B_{12} - S_{55})d_x \quad ; \quad L_{35} = (R^{-1}B_{22} - S_{44})d_y$$

$$L_{44} = D_{11}d_x^2 + D_{66}d_y^2 - S_{55} - Id_t^2 \quad ; \quad L_{45} = (D_{12} + D_{66})d_xd_y$$

$$L_{55} = D_{66}d_x^2 + D_{22}d_y^2 - S_{44} - Id_t^2 \quad ; \quad \bar{C}_\ell = C_\ell/R \quad ; \quad d_x = \partial(\ )/\partial x$$

Appendix B: Thin-Shell Theory

The theory developed here is an extension of the static bimodulus-composite shell analysis presented in Ref. 27. The shell theory used is a generalized first-approximation one that can be reduced to the theories of Sanders, Love's first-approximation, Loo, Morley, and Donnell by use of the tracers listed in Table B-1. It is emphasized that these are *different* tracers than those listed in Table 1.

The shell constitutive equations are the same as given in Eqs. (1)-(3), except now the thickness shear quantities (Q_4 and Q_5) are not needed. The appropriate equations of motion are

$$N_{1,x} + N_{6,y} - (C_4'/R)M_{6,y} = Pu,_{tt}$$

$$N_{6,x} + N_{2,y} + (C_3'/R)M_{6,x} + (C_1'/R)M_{2,y} = Pv,_{tt} \qquad \text{(B1)}$$

$$M_{1,xx} + 2M_{6,xy} + M_{2,yy} - (N_2/R) = Pw,_{tt}$$

The kinematic relations for the middle-surface strains are the same as in Eqs. (6). Since now the thickness shear strains (ε_4 and ε_5) are identically zero, the curvatures are given by

$$\kappa_1 = - w,_{xx} \quad ; \quad \kappa_2 = - w,_{yy} + (C_2'/R)v,_y \qquad \text{(B2)}$$

$$\kappa_6 = - 2w,_{xy} + (C_3'/R)v,_x - (C_4'/R)u,_y$$

Substituting the expressions for the middle-surface strains and curvatures into the constitutive relations and the latter into Eqs. (B1), one obtains the same form of operator expression as Eq. (7) except that now the elements of the $\{\delta\}$ vector are only u, v, w. The linear differential operators are now

$$L_{11} = A_{11}d_x^2 + [A_{66} - 2C_4'B_{66}R^{-1} + (C_4')^2D_{66}R^{-2}]d_y^2 - Pd_t^2$$

$$L_{12} = [A_{12} + A_{66} + C_2'B_{12}R^{-1} + (C_3' - C_4')B_{66}R^{-1} - C_3'C_4'D_{66}R^{-2}]d_xd_y$$

$$L_{13} = - B_{11}d_x^3 - (B_{12} + 2B_{66} - 2C_4'D_{66}R^{-1})d_xd_y^2 + A_{12}R^{-1}d_x = L_{31}$$

$$L_{21} = [A_{12} + A_{66} + C_1'B_{12}R^{-1} + (C_3' - C_4')B_{66}R^{-1} - C_3'C_4'D_{66}R^{-2}]d_xd_y$$

$$L_{22} = [A_{66} + 2C_3'B_{66}R^{-1} + (C_3')^2D_{66}R^{-2}]d_x^2$$
$$+ [A_{22} + (C_1' + C_2')B_{22}R^{-1} + C_1'C_2'D_{22}R^{-2}]d_y^2 - Pd_t^2$$

$$L_{23} = - [B_{12} + 2B_{66} + C_1'D_{12}R^{-1} + C_3'D_{66}R^{-1}]d_x^2d_y$$
$$- (B_{22} + C_1'D_{22}R^{-1})d_y^3 + (A_{22} + C_1'B_{22}R^{-1})R^{-1}d_y$$

$$L_{32} = -[B_{12} + 2B_{66} + C_2'D_{12}R^{-1} + 2C_3'D_{66}R^{-1}]d_x^2d_y$$
$$- (B_{22} + C_2'D_{22}R^{-1})d_y^3 + (A_{22} + C_2'B_{22}R^{-1})R^{-1}d_y$$
$$L_{33} = D_{11}d_x^4 + 2(D_{12} + 2D_{66})d_x^2d_y^2 + D_{22}d_y^4$$
$$- 2B_{12}R^{-1}d_x^2 - 2B_{22}R^{-1}d_y^2 + A_{22}R^{-2} + Pd_t^2 \quad \text{(B3)}$$

Note that the operators L_{rs} are symmetric for all of the theories except Morley's.

The criteria for the neutral-surface locations (z_{nx} and z_{ny}) to be constant are now

$$z_{nx} = u,_x/w,_{xx} = \text{const.}$$
$$z_{ny} = (v,_y + R^{-1}w)/(w,_{yy} - C_2'R^{-1}v,_y) = \text{const.} \quad \text{(B4)}$$

The appropriate boundary conditions for closed cylindrical shells and cylindrically curved panels are the same as Eqs. (11) or Eqs. (12) except that here the conditions on ψ_1 and ψ_2 are not needed (since ψ_1 and ψ_2 do not appear in this thin-shell theory). Similarly, only the first three expressions in Eqs. (13) and the first three elements in Eqs. (15) are required.

Table 2. Geometric Parameters and Material-Property Data for Example I (Ref. 8)

Geometric Parameters	
Wall thickness h, in (mm)	0.02 (0.51)
Middle-surface radius R, in (mm)	2.481 (63.0)
Length L, in (mm)	31.5 (800)
Material Properties (Boron-Epoxy)	
Fiber-direction Young's modulus, psi (GPa)	31.0×10^6 (214)
Transverse Young's modulus, psi (GPa)	2.7×10^6 (18.6)
Major Poisson's ratio, dimensionless	0.28
In-plane and thickness shear moduli, psi (GPa)[a]	0.75×10^6 (5.17)
Specific gravity, dimensionless	2.05

[a] The minor Poisson's ratio is assumed to be given by the reciprocal relation.

Table 3. Natural Frequencies Associated with Various Vibrational Modes for a Complete Cylindrical Shell of Boron-Epoxy (Example I)

No. of axial half waves, m	No. of circum. full waves, n	Natural Frequencies, Hz			
		Ref. 8[a]	Present		
		TST	TST	SDT	SDT wRI
1	1	532	534	533	534
	2	235	241	240	246
	3	253	260	256	257
	4	444	450	418	412
	5	714	721	751	823
	6	1047	1056	1057	1113
	7	1442	1453	1460	1536
	8	1897	1911	1941	2028
2	1	1287	1290	1290	1290
	2	676	683	682	682
	3	443	454	445	444
	4	497	509	480	505
	5	728	739	753	830
	6	1051	1063	1083	1135
	7	1442	1456	1489	1511
	8	1897	1913	1932	2020

[a] The numerical values listed here were taken directly from the computer printout, rather than from the less accurate graphical presentation of Ref. 8, which was not plotted accurately.

TST: thin-shell theory
SDT: shear deformable theory
SDT wRI: shear deformable theory with rotatory inertia

Table 4. Geometric Parameters and Material-Property Data for Example II

Geometric Parameters	Ex. IIA (Ref. 18)	Ex. IIB (Ref. 19)
Panel aspect ratio (L/b)	1	1 to 5
Middle-surface radius/thickness (R/h)	31.25 to 312.5	40
Material Properties		
Fiber-direction Young's modulus	40 E_T	25 E_T
Transverse Young's modulus	E_T	E_T
Major Poisson's ratio[a]	0.25	0.25
In-plane and longitudinal-thickness shear moduli	E_T	0.5 E_T
Transverse-thickness shear modulus	0.5 E_T	0.2 E_T

[a] The minor Poisson's ratio is assumed to be given by the reciprocal relation.

Table 5. Dimensionless Natural Frequencies $\omega b^2(P/E_T h^3)^{1/2}$ Associated with Various Vibrational Modes for Cylindrically Curved Panels (Example IIA)

No. of axial half waves m	No. of circ. half waves n	b/h	R/h	w/o Coupling (B_{ij}=0) Ref. 18	w/o Coupling (B_{ij}=0) Present	w/Coupling ($B_{ij}\neq0$) Ref. 18	w/Coupling ($B_{ij}\neq0$) Present
1	1	50	312.5	24.76	20.59	19.23	14.08
2	1	50	312.5	53.44	55.40	-	-
1	2	50	312.5	60.43	53.58	-	-
2	2	50	312.5	75.86	75.51	-	-
1	1	50	156.25	36.00	24.48	32.86	19.32
2	1	50	156.25	54.91	60.89	-	-
1	2	50	156.25	78.74	53.95	-	-
2	2	50	156.25	80.34	76.66	-	-
1	1	10	62.5	22.00	16.91	18.57	11.34
2	1	10	62.5	35.06	38.30	-	-
1	2	10	62.5	45.01	38.19	-	-
2	2	10	62.5	48.46	51.70	-	-
1	1	10	31.25	34.86	17.11	32.38	11.64
2	1	10	31.25	37.27	38.63	-	-
1	2	10	31.25	67.63	38.21	-	-
2	2	10	31.25	55.21	51.77	-	-

Table 6. Dimensionless Fundamental Frequencies $\omega b^2(P/E_T h^3)^{1/2}$ for Cylindrically Curved Panels (Example IIB)

Panel Aspect Ratio L/b	Without Coupling		With Coupling	
	Ref. 19	Present	Ref. 19	Present
1	16.11	15.89	11.71	11.65
2	10.94	10.79	7.35	7.37
3	10.27	10.10	6.58	6.59
4	10.07	9.91	6.32	6.33
5	9.99	9.83	6.22	6.21

Table 7. Material Properties for Aramid-Rubber Bimodulus Composite Material[a]

Property	k = 1	k = 2
Fiber-direction Young's modulus, GPa	3.58	0.0120
Transverse Young's modulus, GPa	0.00909	0.0120
Major Poisson's ratio, dimensionless	0.416	0.205
Minor Poisson's ratio, dimensionless	0.00106	0.205
In-plane and longitudinal-thickness shear moduli, GPa	0.00370	0.00370
Transverse-thickness shear modulus, GPa	0.00290	0.00499
Specific gravity, dimensionless	0.970	

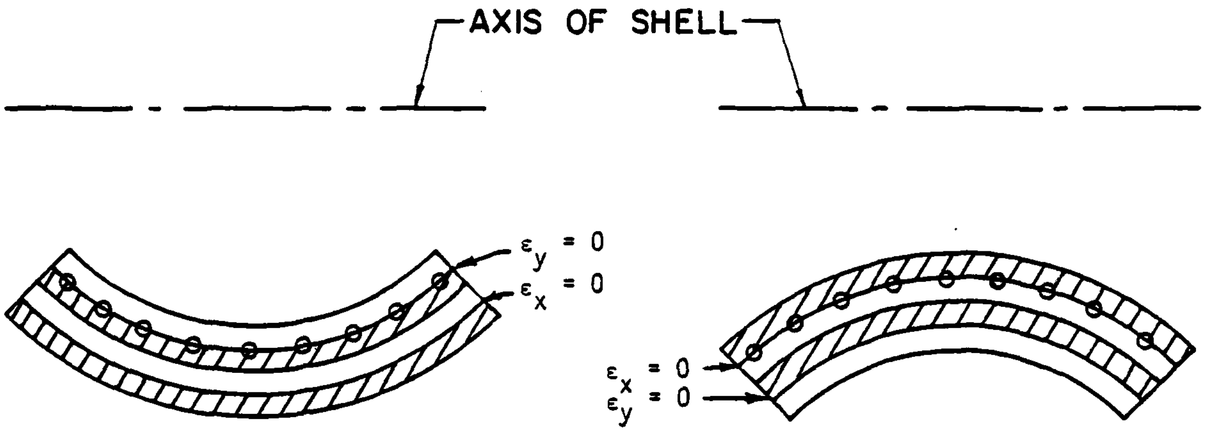
[a] Fiber-direction tension is denoted by k = 1, and fiber-direction compression by k = 2.

Table 8. Dimensionless Neutral-Surface Locations in the First and Second Portions of a Cycle for Two-Layer, Cross-Ply Cylindrical Panels Constructed of Aramid-Rubber (Sanders-Type Theory with Thickness Shear Deformation; b/h = 10)

Aspect ratio a/b	First Portion		Second Portion	
	$Z_x^{(1)}$	$Z_y^{(1)}$	$Z_x^{(2)}$	$Z_y^{(2)}$
	R/h = 10 (Highly Curved)			
0.5	0.4405	-0.0987	-0.0272	0.3824
0.7	0.4343	-0.0736	-0.0374	0.4108
1.0	0.4234	-0.0521	-0.0546	0.4283
1.4	0.4069	-0.0368	-0.0768	0.4378
2.0	0.3796	-0.0250	-0.1101	0.4433
	R/h = 20 (Moderately Curved)			
0.5	0.4430	-0.0827	-0.0226	0.4033
0.7	0.4387	-0.0618	-0.0316	0.4220
1.0	0.4312	-0.0438	-0.0459	0.4338
1.4	0.4195	-0.0311	-0.0648	0.4401
2.0	0.3995	-0.0213	-0.0930	0.4439
	R/h = 50 (Slightly Curved)			
0.5	0.4446	-0.0719	-0.0193	0.4161
0.7	0.4418	-0.0539	-0.0272	0.4289
1.0	0.4361	-0.0384	-0.0394	0.4371
1.4	0.4276	-0.0273	-0.0558	0.4414
2.0	0.4130	-0.0190	-0.0801	0.4441

Table B-1 Thin-shell-theory tracers and their values

Theory	C_1'	C_2'	C_3'	C_4'
Sanders	1	1	3/2	1/2
Love's first approximation	1	1	1	0
Loo	1	0	1	0
Morley	1	0	0	0
Donnell	0	0	0	0



(a) First portion of cycle

(b) Second portion of cycle

Fig. 2 Bimodulus action during the two portions of motion of a two-layer, cross-ply shell in the fundamental mode of vibration. Inside layer is in y direction (90°); outside layer is in x direction (0°). Shaded portions are in tension in the respective fiber directions.

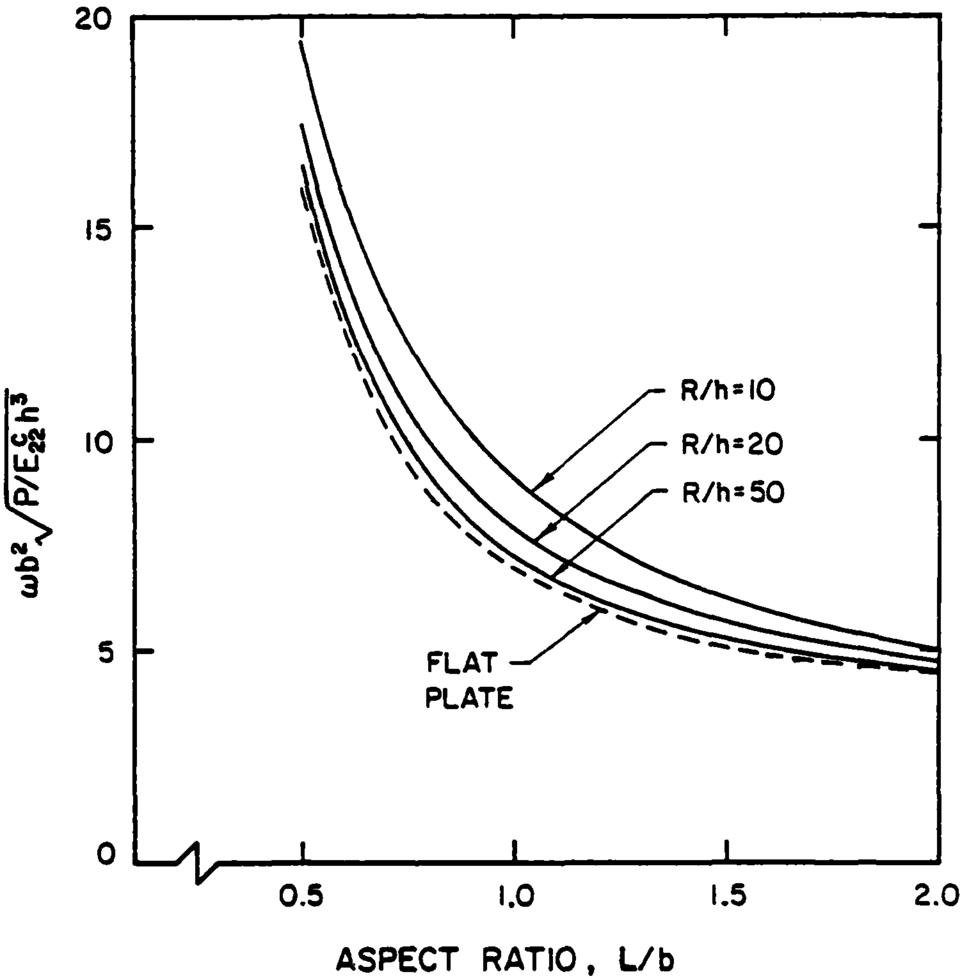


Fig. 3 Variation of fundamental frequency with aspect ratio for two-layer, cross-ply curved panels of aramid-rubber.

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A theory is formulated for the small-amplitude free vibration of thick, circular cylindrical shells laminated of bimodulus composite materials, which have different elastic properties depending upon whether the fiber-direction strain is tensile or compressive. The theory used is the dynamic, shear deformable (moderately thick shell) analog of the Sanders best first-approximation thin-shell theory. By means of tracers, the analysis can be reduced to various simpler shell theories, namely Love's first approximation,
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20. Abstract - Cont'd

and Donnell's shallow-shell theory. As an example of the application of the theory, a closed-form solution is presented for a freely supported panel or complete shell. To validate the analysis, numerical results are compared with existing results for various special cases. Also, the effects of the various shell theories, thickness shear flexibility, and bimodulus action are investigated.

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